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Q1) Find the Future value and compound interest on an investment of 700 \$ compounded quarterly at 10% for 3 years.

Solution:

$$FV = S = P(1+r)^N$$

Since $P=700$ \$, $N=n*T=3*4=12$, $r=R/n=0.1/4=0.025$
 $FV = S = P(1+r)^N = 700(1+0.025)^{12} = 700*1.34489 = 941.42$ \$

compound interest = $I = FV - P = 941.42 - 700 = 241.42$ \$.

Q2) Mr X deposited 4000 \$ in a saving account paying 2% interest compounded daily. Find Future value of the money and compound interest he earned at the end of 5 years.

Solution:

$$FV = S = P(1+r)^N$$

Since $P=4000$ \$, $N=n*T=5*365=1825$,
 $r=R/n=0.02/365=0.025=5.48*10^{-5}$
 $FV = S = P(1+r)^N = 4000(1+5.48*10^{-5})^{1825} = 4000*1.10517 = 4420.67$ \$
 compound interest = $I = FV - P = 4420.67 - 4000 = 420.67$ \$.

What principle will amount to \$3000 if invested at 6.5% compounded weekly for 4 years?

$$A = 3000, r = 0.065, n = 52, t = 4 \quad \text{Identify each variable}$$

$$3000 = P \left(1 + \frac{0.065}{52} \right)^{52 \cdot 4} \quad \text{Evaluate parentheses}$$

$$3000 = P(1.00125)^{52 \cdot 4} \quad \text{Multiply exponent}$$

$$3000 = P(1.00125)^{208} \quad \text{Evaluate exponent}$$

$$3000 = P(1.296719528...) \quad \text{Divide each side by 1.296719528...}$$

$$\frac{3000}{1.296719528...} = \frac{P}{1.296719528...}$$

$$2313.53 = P \quad \text{Solution for } P$$

$$\text{\$}2313.53 \quad \text{Our Solution}$$

Q3

Q4

If you take a car loan for \$25000 with an interest rate of 6.5% compounded quarterly, no payments required for the first five years, what will your balance be at the end of those five years?

$$P = 25000, r = 0.065, n = 4, t = 5 \quad \text{Identify each variable}$$

$$A = 25000 \left(1 + \frac{0.065}{4} \right)^{4 \cdot 5} \quad \text{Plug each value into formula, evaluate parenthesis}$$

$$A = 25000(1.01625)^{20} \quad \text{Multiply exponents}$$

$$A = 25000(1.01625)^{20} \quad \text{Evaluate exponent}$$

$$A = 25000(1.38041977...) \quad \text{Multiply}$$

$$A = 34510.49$$

$$\text{\$}34,510.49 \quad \text{Our Solution}$$

If \$4000 is invested in an account paying 3% interest compounded daily, what is the balance after 7 years?

$$P = 4000, r = 0.03, n = 365, t = 7 \quad \text{Identify each variable}$$

$$A = 4000 \left(1 + \frac{0.03}{365} \right)^{365 \cdot 7} \quad \text{Plug each value into formula, evaluate parenthesis}$$

$$A = 4000(1.00008219...)^{365 \cdot 7} \quad \text{Multiply exponent}$$

$$A = 4000(1.00008219...)^{2555} \quad \text{Evaluate exponent}$$

$$A = 4000(1.23366741...) \quad \text{Multiply}$$

$$A = 4934.67$$

$$\text{\$4934.67} \quad \text{Our Solution}$$

While this difference is not very large, it is a bit higher. The table below shows the result for the same problem with different compounds.

Compound	Balance
Annually	\\$4919.50
Semi-Annually	\\$4927.02
Quarterly	\\$4930.85
Monthly	\\$4933.42
Weekly	\\$4934.41
Daily	\\$4934.67

If \$4000 is invested in an account paying 3% interest compounded monthly, what is the balance after 7 years?

$$P = 4000, r = 0.03, n = 12, t = 7 \quad \text{Identify each variable}$$

$$A = 4000 \left(1 + \frac{0.03}{12} \right)^{12 \cdot 7} \quad \text{Plug each value into formula, evaluate parentheses}$$

$$A = 4000(1.0025)^{12 \cdot 7} \quad \text{Multiply exponents}$$

$$A = 4000(1.0025)^{84} \quad \text{Evaluate exponent}$$

$$A = 4000(1.2333548) \quad \text{Multiply}$$

$$A = 4933.42$$

$$\text{\$4933.42} \quad \text{Our Solution}$$

Q7) If you deposit \$4000 into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years?

Solution:

$$FV = 4000 \left(1 + \frac{0.06}{4} \right)^{4(5)}$$

Plug in the giving information, $P = 4000$, $r = 0.06$, $n = 4$, and $t = 5$.

$$FV = 4000(1.015)^{20}$$

Use the order or operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 4000(1.346855007)$$

$$FV = 5387.42$$

Round your final answer to two decimals places.

After 5 years there will be \$5387.42 in the account.

Example 6: At 3% annual interest compounded monthly, how long will it take to double your money?

At first glance it might seem that this problem cannot be solved because we do not have enough information. It can be solved as long as you double whatever amount you start with. If we start with \$100, then $P = \$100$ and $FV = \$200$.

$$200 = 100 \left(1 + \frac{0.03}{12} \right)^{12t}$$

Plug in the giving information, $FV = 200$, $P = 100$, $r = 0.03$, and $n = 12$.

$$200 = 100(1.0025)^{12t}$$

Use the order of operations to simplify the problem. Keep as many decimals as possible until the final step.

$$2 = 1.0025^{12t}$$

Divide each side by 100.

$$\log(2) = \log(1.0025^{12t})$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\log 2 = (12t)(\log 1.0025)$$

$$\frac{\log 2}{\log 1.0025} = 12t$$

Divide each side by $\log 1.0025$.

$$277.6053016 \approx 12t$$

Use a calculator to find $\log 2$ divided by $\log 1.0025$.

$$t \approx 23.1$$

Finish solving the problem by dividing each side by 12 and round your final answer.

At 3% annual interest it will take approximately 23.1 years to double your money.

Example 4: If you deposit \$5000 into an account paying 6% annual interest compounded monthly, how long until there is \$8000 in the account?

$$8000 = 5000 \left(1 + \frac{0.06}{12} \right)^{12t}$$

Plug in the giving information, $FV = 8000$, $P = 5000$, $r = 0.06$, and $n = 12$.

$$8000 = 5000(1.005)^{12t}$$

Use the order of operations to simplify the problem. Keep as many decimals as possible until the final step.

$$1.6 = 1.005^{12t}$$

Divide each side by 5000.

$$\log(1.6) = \log(1.005^{12t})$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\log 1.6 = (12t)(\log 1.005)$$

$$\frac{\log 1.6}{\log 1.005} = 12t$$

Divide each side by $\log 1.005$.

$$94.23553232 \approx 12t$$

Use a calculator to find $\log 1.6$ divided by $\log 1.005$.

$$t \approx 7.9$$

Finish solving the problem by dividing each side by 12 and round your final answer.

It will take approximately 7.9 years for the account to go from \$5000 to \$8000.

Example 5: If you deposit \$8000 into an account paying 7% annual interest compounded quarterly, how long until there is \$12400 in the account?

$$12400 = 8000 \left(1 + \frac{0.07}{4} \right)^{4t}$$

Plug in the giving information, $FV = 12400$, $P = 8000$, $r = 0.07$, and $n = 4$.

$$12400 = 8000(1.0175)^{4t}$$

Use the order of operations to simplify the problem. Keep as many decimals as possible until the final step.

$$1.55 = 1.0175^{4t}$$

Divide each side by 8000.

$$\log(1.55) = \log(1.0175^{4t})$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\log 1.55 = (4t)(\log 1.0175)$$

$$\frac{\log 1.55}{\log 1.0175} = 4t$$

Divide each side by $\log 1.0175$.

$$25.26163279 \approx 4t$$

Use a calculator to find $\log 1.55$ divided by $\log 1.0175$.

$$t \approx 6.3$$

Finish solving the problem by dividing each side by 4 and round your final answer.

It will take approximately 6.3 years for the account to go from \$8000 to \$12400.

Example 2: If you deposit \$6500 into an account paying 8% annual interest compounded monthly, how much money will be in the account after 7 years?

$$FV = 6500 \left(1 + \frac{0.08}{12} \right)^{12(7)}$$

Plug in the giving information, $P = 6500$, $r = 0.08$, $n = 12$, and $t = 7$.

$$FV = 6500(1.00666666)^{84}$$

Use the order of operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 6500(1.747422051)$$

$$FV = 11358.24$$

Round your final answer to two decimals places.

After 7 years there will be \$11358.24 in the account.

Example 3: How much money would you need to deposit today at 9% annual interest compounded monthly to have \$12000 in the account after 6 years?

$$12000 = P \left(1 + \frac{0.09}{12} \right)^{12(6)}$$

Plug in the giving information, $FV = 12000$, $r = 0.09$, $n = 12$, and $t = 6$.

$$12000 = P(1.0075)^{72}$$

Use the order of operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$12000 = P(1.712552707)$$

$$P = 7007.08$$

Divide and round your final answer to two decimals places.

You would need to deposit \$7007.08 to have \$12000 in 6 years.

Q13) How long it will take unit your 50000 saving grow to 100000 \$ at 3%.

Solution: $FV=(1+r)^N$

$$100000=50000(1+0.03)^n$$

$$2= (1+0.03)^n$$

$$\ln= n\ln(1+0.03)$$

$$N=\ln 2/\ln(1.03)=0.69315/0.02956=23.45=24 \text{ year.}$$

Q)A machine is purchased (plant-and-equipment investment)at a cost of 150000 \$. The machine has a residual book value of 10000 \$ and a useful life of 10 years(depreciable years). IF the machine is used for four month in the initial year of poaches, what is the depreciation expense for the third year?

Solution: $BV=p(1-r)^n$

$$10000=150000 (1-r)^{10}$$

$$(1-r)^{10}=10000/150000=1/15$$

$$(1-r) =(1/15)^{1/10}$$

$$(1-r) =(1/15)^{0.1}=(0.066)^{0.1}=0.762$$

$$r=1-0.762=0.237 \text{ or } r=0.237*100=23.7$$

Q14) Suppose that Ahmed deposits 2000 \$ every 6 month in a saving account which pays 12% annual interest compounded half-yearly. How much will Ahmed have in the bank at the end of 8 years. Assuming that Ahmed deposits his money at the beginning of each 6-month period.

Q15) What is the effective annual rate (EAR) of 6.5% simple nominal annual rate compounded monthly?

Solution:

$$E=(1+r)^N-1=(1+(.065/12))^{12}-1=6.697\%$$

Q16) Find the effective rates if 9% compounded semiannually

Solution:

$$\text{Effective rates } = E = (1+r)^n - 1$$

$$N=n*T=1*2=2, \quad r=0.09/2=0.045$$

$$E=(1+r)^n-1=(1+0.045)^2-1=0.092=9.2\%.$$

Q 17) Find the effective rates if 8% compounded daily

Solution:

$$\text{Effective rates } = E = (1+r)^n - 1$$

$$N=n*T=1*365=365, \quad r=0.08/365=2.192*10^{-4}=0.0002192$$

$$E=(1+r)^n-1=(1+0.0002192)^{365}-1=0.083=8.3\%.$$

Q18) Find the yearly depreciation of a loan tractor costing 1800 \$. It has an expected lifetime of 5 years and a scrap value of 200\$. Using the straight Line Method depreciation method.

Solution:

$$\text{Depreciation } D = \frac{\text{Original cost or Replacement Value} - \text{Scrap value or Salvage value}}{\text{life of the property in years}} = \frac{C-S}{n}$$

$$\text{Deprecation} = (1800-200)/5 = 320 \$ \text{ yearly depreciation.}$$

Q 19) The Eastern company provides the following information regarding one of its fixed assets that has been purchased on January 1, 2015:

Cost of the asset	\$35,000
Salvage value	\$3,000
Useful life	10 years

Find the annual depreciation expense of this asset using straight line method.

Solution:

$$\begin{aligned}\text{Deprecation} &= (35,000 - 3,000)/10 \\ &= 3,200 \$.\end{aligned}$$

Q 20) An argon gas processor has a first cost of \$20,000 with a \$5,000 salvage value after 5 years. Find the annual Depreciation expense of this asset using straight line method.

Solution:

$$\begin{aligned}\text{precaution} &= (C - S)/n \\ &= (20,000 - 5,000)/5 \\ &= 3,000 \$\end{aligned}$$

Q21) Compute the accumulated value of an annuity of 100 \$ invested at the end of each month for one year at an annual rate of 6%.

Solution:

$$FV_{\text{AnnuityOrdinary}} = PMT \frac{\left[\left(1 + \frac{R}{n} \right)^{nt} - 1 \right]}{R}$$

$$FV_{\text{AnnuityOrdinary}} = 100 \times \frac{\left[\left(1 + \frac{0.06}{12} \right)^{12 \times 1} - 1 \right]}{\frac{0.06}{12}}$$

$$FV = 100 \times 12.33556 = 1233.56 \$$$

Q 22) Suppose that Ahmed deposits 1000 \$ every 3 months in saving account which pays 8% annual interest compounded quarterly. How much will Ahmed have in the bank at the end of 5 years?

Solution: $FV_{\text{AnnuityOrdinary}} = PMT \times \frac{\left[\left(1 + \frac{R}{n} \right)^{nt} - 1 \right]}{R}$

$$FV_{\text{AnnuityOrdinary}} = 1000 \times \frac{\left[\left(1 + \frac{0.08}{4} \right)^{5 \times 4} - 1 \right]}{\frac{0.08}{4}}$$

$$FV = 1000 \times 24.29737 = 24297.37 \$$$

Q 23) Suppose you make regular deposits of \$75 in an ordinary annuity account that gives 7.2% annual interest rate compounded monthly for 10 years. Find the following:

- The future value of the account (at the end of the accumulation period).
- The total of all deposits.
- The total interest earned.

Solution:

$$\mathbf{FV}_{\text{AnnuityOrdinary}} = \mathbf{PMT} \frac{\left[\left(1 + \frac{\mathbf{R}}{\mathbf{n}} \right)^{\mathbf{nt}} - 1 \right]}{\frac{\mathbf{R}}{\mathbf{n}}}$$

$$\text{a) } V = \frac{75 \left[\left(1 + \frac{.072}{12} \right)^{10 \cdot 12} - 1 \right]}{\frac{.072}{12}} = \$13,125.23$$

b) To find the total of all deposits, multiply the amount of the regular deposit by the total number of deposits, $10 \cdot 12 = 120$.

$$120(\$75) = \$9,000$$

c) To find the total interest earned, subtract the total deposits from the total accumulation:

$$13125.23 - 9000 = \$4125.23$$

Q 24) The owners of n tool and die company wants to accumulate 50,000 to replace worn-out equipment in 8 year from now. How much should they contribute each month into a sinking fund which pay 8% compounded quarterly?

Solution:

$$\mathbf{FV}_{\text{AnnuityOrdinary}} = \mathbf{PMT} \frac{\left[\left(1 + \frac{\mathbf{R}}{\mathbf{n}} \right)^{\mathbf{nt}} - 1 \right]}{\frac{\mathbf{R}}{\mathbf{n}}}$$

$$50,000 = \text{PMT} * \frac{\left[\left(1 + \frac{0.08}{4} \right)^{8 \times 12} - 1 \right]}{\frac{0.08}{4}}$$

$$50,000 = \text{PMT} * \frac{[(1 + 0.02)^{96} - 1]}{0.02}$$

$$\text{PMT} = 50,000 / 284.34666 = 175.66 \text{ monthly payment.}$$

Problem 1. Suppose you deposit \$900 per month into an account that pays 4.8% interest, compounded monthly. How much money will you have after 9 months?

Solution: We want to know how much we will have in the future, so we use the formula for the future value of a sinking fund:

$$FV = \text{PMT} \frac{(1+i)^n - 1}{i}$$

In this case $i = \frac{0.048}{12} = 0.0004$ and $n = 12 * \left(\frac{9}{12} \right) = 9$ (note that 9 months is $t = \frac{9}{12}$ of a year). Thus, the future value is

$$FV = 900 * \frac{(1 + 0.0004)^9 - 1}{0.0004} = 900 * \frac{(1.0004)^9 - 1}{0.0004} = 8112.97$$

Solution:

$$\text{FV}_{\text{AnnuityDue}} = \text{PMT} \frac{\left[\left(1 + \frac{\text{R}}{\text{n}} \right)^{\text{nt}} - 1 \right]}{\frac{\text{R}}{\text{n}}} \times \left(1 + \frac{\text{R}}{\text{n}} \right)$$

$$\text{FV}_{\text{AnnuityDue}} = 20000 \frac{\left[\left(1 + \frac{0.12}{2} \right)^{8 \times 2} - 1 \right]}{\frac{0.12}{2}} \times \left(1 + \frac{0.12}{2} \right)$$

$$\text{FV}_{\text{AnnuityDue}} = 20000 \frac{[(1 + 0.06)^{16} - 1]}{0.06} \times (1 + 0.06)$$

$$FV_{\text{AnnuityDue}} = 20000 * 25.67253 * 1.06 = 54425.76\$$$

Q26) Ahmed purchases a life insurance policy which has an annual premium of 1500 \$ due at the beginning of the year. If he elects to pay his premium in quarterly installments, how much should he pay at the beginning of each quarter if the interest rate is 10% compounded quarterly.

Solution:

$$FV_{\text{AnnuityDue}} = PMT \frac{\left[\left(1 + \frac{R}{n} \right)^{nt} - 1 \right]}{\frac{R}{n}} * \left(1 + \frac{R}{n} \right)$$

$$1500 = PMT * \frac{\left[\left(1 + \frac{0.1}{4} \right)^{nt} - 1 \right]}{\frac{0.1}{4}} * \left(1 + \frac{0.1}{4} \right)$$

$$1500 = PMT * 4.25633$$

$$PMT = 1500 / 4.25633 = 352.42 \$ \text{ compounded quarterly.}$$

Q 27) Mr Smith would like to receive 40000 \$ each quarter for 10 years after he retires. How much money (to nearest dollar) dose he have to save in a money market fund which pays at the rate of 8% compounded quarterly.

Solution:

$$PV(\text{Present Value}) = PMT * \left[\frac{(1 - (1+r)^{-N})}{r} \right]$$

$$PV = 4000 * \left[\frac{(1 - (1 + \frac{0.08}{4})^{-10 \times 4})}{\frac{0.08}{4}} \right]$$

$$PV = 4000 * 27.35548 = 109421.92 \$.$$

Q 28) Find the future value of an investment if \$150 is deposited at the beginning of each month for 9 years and the interest rate is 7.2%, compounded monthly.

Solution:

$$FV_{\text{AnnuityDue}} = 100 \left[\frac{\left(1 + \frac{0.072}{4}\right)^{nt} - 1}{\frac{0.072}{4}} \right] * \left(1 + \frac{0.072}{4}\right)$$

$$FV_{\text{AnnuityDue}} = 22,836.59\$$$

Q29) Chase Bank in April 2015 advertised a new car auto loan rate of 2.23% for a 48-month loan. Shelley Fasulko will buy a new car for \$25,000 with a down payment of \$4500. Find the amount of each payment.

Solution: After a \$4500 down payment, the loan amount is (25,000 - 4500 = \$20,500. Use the present value formula for an annuity, with p = 20, 500, n = 48, and r = .0223/12 (the monthly interest rate). Then solve for payment PMT. with P = 20, 500, n = 48, and r = .0223/12 (the monthly interest rate). Then solve for payment PMT.

$$PV(\text{Present Value}) = PMT * \left[\frac{(1 - (1+r)^{-N})}{r} \right]$$

$$20,500 = PMT * \left[\frac{(1 - (1 + 0.0223/12)^{-48})}{0.0223/12} \right]$$

$$PMT = \$446.81.$$

Q 30) Suzan has saved a total of 170,000 \$ for retirement. She has put the money in a fund which pays 9% annual interest compounded monthly. How much should she withdraw each month in order to have enough money to last for 15 years?

Solution:

$$PV(\text{Present Value}) = PMT * \left[\frac{(1 - (1 + r)^{-N})}{r} \right]$$

$$170,000 = PMT * \left[\frac{(1 - (1 + 0.09/12)^{-15 \times 12})}{0.09/12} \right]$$

$$170,000 = PMT * (0.73945 / 0.0075)$$

$$PMT = 170,000 / 98.59341 = 1,724.25 \text{ Monthly payment}$$

Q31) A person is saving money each week in order to accumulate a 25,000 \$ down payment for a house. How much must this person deposit weekly into an account paying 13% compounded weekly if the down payment is needed in 5 years.

Solution:

$$FV_{\text{AnnuityOrdinary}} = PMT \frac{\left[\left(1 + \frac{R}{n} \right)^{nt} - 1 \right]}{\frac{R}{n}}$$

$$25,000 = PMT \frac{\left[\left(1 + \frac{0.13}{52} \right)^{5 \times 52} - 1 \right]}{\frac{0.13}{52}}$$

$$25,000 = \text{PMT} * 365.5950$$

$$\text{PMT} = (25,000) / (365.5950) = 68.38 \text{ weekly deposited.}$$

Q32) Suppose you win a lottery that entitles you to receive 800 \$ per month for the next 30 years. If money is worth 6% compounded monthly. What is the present value of this annuity?

Solution:

$$\text{PV(Present Value)} = \text{PMT} * \left[\frac{(1 - (1 + r)^{-N})}{r} \right]$$

$$\text{PV(Present Value)} = 800 * \left[\frac{(1 - (1 + 0.06/12)^{-30 \times 12})}{0.06/12} \right]$$

$$\text{PV(Present Value)} = 800 * 166.79161 = 133,433.29 \$$$

Q33) In order to buy a new house, a family obtains a mortgage of 220,000 \$ for 30 years at 6.9% compounded monthly. What are the monthly payments?

Solution: $\text{PV(Present Value)} = \text{PMT} * \left[\frac{(1 - (1 + r)^{-N})}{r} \right]$

$$220,000 = \text{PMT} * \left[\frac{(1 - (1 + 0.069/12)^{-30 \times 12})}{0.069/12} \right]$$

$$220,000 = \text{PMT} * \left[\frac{(1 - (1 + 0.00575)^{-360})}{0.00575} \right]$$

$$\text{PMT} = (220000) / (151.83720) = 1,448.92 \text{ monthly payment}$$

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Q34) Ahmed purchased a house for 350,000 \$. He made 20% down payment and obtained a mortgage at an annual rate of 8.4% for 20 years, find:

- The amount of the down payment.
- The amount of the mortgage.
- The monthly payment.
- The amount of interest at the first month.

Solution:

a. $350,000 \times (20/100) = 70,000$ \$ down payment.

b. $350,000 - 70,000 = 280,000$ \$ the amount of the mortgage.

c. $PV(\text{Present Value}) = PMT \times \left[\frac{(1 - (1+r)^{-N})}{r} \right]$

$$280,000 = PMT \times \left[\frac{(1 - (1 + 0.084/12)^{-20 \times 12})}{0.084/12} \right]$$

$$280,000 = PMT \times \left[\frac{(1 - (1 + 0.007)^{-240})}{0.007} \right]$$

$$PMT = (280,000 / 116.0766) = 2412.21 \$$$

d. $I = PRT = 280,000 \times 0.084 \times (1/12) = 1960 \$$.

Q35) Beth has just received an inheritance of \$400,000 and would like to be able to make monthly withdrawals over the next 15 years. She decides on an annuity that pays 6.7%, compounded monthly. How much will her monthly payments be in order to draw the account down to zero at the end of 15 years?

Solution:

$$PV(\text{Present Value}) = PMT * \left[\frac{(1 - (1+r)^{-N})}{r} \right]$$

$$400000 = PMT * \left[\frac{(1 - (1 + 0.067/12)^{-12*15})}{0.067/12} \right]$$

$$PMT = 3528.56 \$.$$

Thus, each withdrawal will be \$3,528.56. At the end of the 15 years, nothing will be left.

Q36)

a. What is the future value of 25 \$ deposited monthly for 30 years at a rate of 2% compounded monthly?

b. What is amount of interest earned on this investment?

Solution:

a.

$$FV = PMT * \frac{\left[\left(1 + \frac{R}{n} \right)^{nt} - 1 \right]}{\frac{R}{n}}$$

$$FV = 25 * \frac{\left[\left(1 + \frac{0.02}{12} \right)^{12*30} - 1 \right]}{\frac{0.02}{12}} = 12,318.14$$

b. Interest= Ending Balance- Total Payment

$$\text{Interest} = 12,318.14 - 30 * 12 * 25 = 12,318.14 - 9000 = 3318.14 \$.$$

Problem 1. Suppose you deposit \$900 per month into an account that pays 4.8% interest, compounded monthly. How much money will you have after 9 months?

Solution: We want to know how much we will have in the future, so we use the formula for the future value of a sinking fund:

$$FV = PMT \frac{(1 + i)^n - 1}{i}$$

In this case $i = \frac{0.048}{12} = 0.0004$ and $n = 12 * \left(\frac{9}{12}\right) = 9$ (note that 9 months is $t = \frac{9}{12}$ of a year). Thus, the future value is

$$FV = 900 * \frac{(1 + 0.0004)^9 - 1}{0.0004} = 900 * \frac{(1.0004)^9 - 1}{0.0004} = 8112.97$$

Q 38) *You have a retirement account with \$2000 in it. The account earns 6.2% interest, compounded monthly, and you deposit \$50 every month for the next 20 years. How much will be in the account at the end of those 20 years?*

Solution: A retirement account is a sinking fund since you are making periodic deposits. In this case, there's already \$2,000 in the account when you start making the periodic deposits. We need to treat this original \$2,000 separately – we will treat it as a separate account that earns compound interest. The formula for the future value of an account that earns compound interest is

$$FV = PV * \left(1 + \frac{r}{m}\right)^{mt}$$

For this formula, m is the number of times compounded per year (12 in this case since it's compounded monthly). So in 20 years, the \$2,000 that was already in the account will be worth

$$FV = 2000 * \left(1 + \frac{0.062}{12}\right)^{12*20} = 6889.20$$

Now the \$50 per month for 20 years is the sinking fund part, so we use the future value of a sinking fund formula to see how much that will be worth 20 years from now:

$$FV = 50 * \frac{\left(1 + \frac{0.062}{12}\right)^{12*20} - 1}{\frac{0.062}{12}} = 23657.42$$

The total amount in the account after 20 years will be the sum of what we got from the original \$2,000 and the total amount from our monthly deposits: $\$6,889.20 + \$23,657.42 = \$30,546.62$.

